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This programed mathematics textbook (Volume I) is for student use in vocational education courses. It was developed as part of a programed series covering 21 mathematical competencies which were identified by university researchers through task analysis of several occupational clusters. The development of a sequential content structure was also based on these mathematics competencies. After completion of this program the student should be able to: (1) recognize a correct equation of the type  $a=bc$ , where  $a$ ,  $b$ ,  $c$  are either letters or positive integers less than 100, (2) recognize equivalent statements of the general equation  $a=bc$ , when these statement are obtained by replacement, multiplication, or division, (3) select the correct method (replacement, multiplication, or division) for deriving an equivalent statement from an equation of the form  $a=bc$ , (4) demonstrate competency in the previous objectives by correctly answering four out of five multiple choice test items covering each objective. The material is to be used by individual students under teacher supervision. Twenty-six other programed texts and an introductory volume are available as VT 006 882-VT 006 909, and VT 006 975. (EM)

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FINAL REPORT  
Project No. OE7-0031  
Contract No. OEG-4-7-070031-1626  
Report No. 16-N

Occupational Mathematics  
EQUIVALENT FORMS OF  $A = BC$

June 1968

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Occupational Mathematics

EQUIVALENT FORMS OF  $A = BC$

Project No. OE7-0031  
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Report No. 16-N

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State Coordinating Council for Occupational Education, Olympia, Washington

### OBJECTIVES

1. The student should be able to demonstrate a recognition of a correct equation of the type  $a = bc$ , where  $a, b, c$  are either letters or positive integers less than 100.
2. The student should be able to demonstrate a recognition of equivalent statements of the general equation  $a = bc$ , when these statements are obtained by replacement, multiplication or division.
3. The student should be able to select the correct method (replacement, multiplication or division) for deriving an equivalent statement from an equation of the form  $a = bc$ .

The student should be able to demonstrate that he has attained competency in the four previously mentioned objectives by correctly answering 4 out of 5 multiple-choice test items covering each objective.

Greetings! You are about to begin improving your knowledge of basic mathematics. There are many important uses for the mathematics you are learning.

This booklet is not like your ordinary books. It is designed to help you learn as an individual. On the following pages you will find some information about mathematics. After the information is presented you will be asked a question. Your answers to these questions will determine how you proceed through this booklet. When you have selected your answer to the question, turn to the page you are told to.

Do not write in this booklet. You may wish to have a pencil and some paper handy so you can write when you want to.

Remember this is not an ordinary book.

1. Study the material on the page.
2. Read the question on the page (you may want to restudy the material on the page).
3. Select the answer you believe is correct.
4. Turn to the page indicated by your answer.

Are you ready to begin?

- (a) Yes      Turn to page 1
- (b) No        Turn to page C
- (c) HELP     Go see your teacher

Page C

Your answer was (b) No.

Well, this booklet is a little different.

Go back and read page B again. After you have read it  
you will probably be ready to begin.

An equation is a statement that two amounts are equal. For example, the quantity  $5 + 2$  is equal to 7, so we write  $5 + 2 = 7$ . Equations are often used to show relationship between numbers. The equation  $5 + 2 = 7$  was used to show that the numerals 2, 5 and 7 are related. The equation  $5 = 7 - 2$  also shows the relationship between 2, 5 and 7. We can show the relationship between 2, 5 and 10 with the equation  $10 = 5 \times 2$ . This simply means that 10 is equal to 5 multiplied by 2.

Sometimes in mathematics we substitute letters for numbers. We can show the relationship between F, M and A with the equation  $F = MA$ . The statement  $F = MA$  means that F is equal to M multiplied by A.

Which of the following is an equation which shows a correct relationship?

- (a)  $3 + 4 + 7$       Turn to page 8
- (b)  $5 \times 2 + 3$       Turn to page 8
- (c)  $3 + 5 = 8$       Turn to page 4

Your answer was  $12/4 = 3$ . Very good!

Which of the following equations is equivalent to  
 $a = bc$ ?

(a)  $a/c = b$

Turn to page 6

(b)  $c/a = b$

Turn to page 28

(c)  $ab = c$

Turn to page 28

(d) I am not sure what equivalent equations are.  
Turn to page 28



Let's try one more.

Which of the following equations is equivalent to  
 $a = bc$ ?

- |  |                 |
|--|-----------------|
| (a) $a/c = b$                                    | Turn to page 6  |
| (b) $c/a = b$                                    | Turn to page 28 |
| (c) $ab = c$                                     | Turn to page 28 |
| (d) I am not sure what equivalent equations are. | Turn to page 28 |

Your answer was (c)  $3 + 5 = 8$ . Correct! This equation expresses the relation that 3 plus 5 equals 8.

Which of the following is a correct equation?

- (a)  $x = wp$       Turn to page 5
- (b)  $x + w + p$       Turn to page 23
- (c)  $3 \times 4 + 7$       Turn to page 19

Page 5

Your answer was (a)  $x = wp$ . Very good! The equation  $x = wp$  expresses the relationship between  $x$ ,  $w$  and  $p$  which says  $x$  equals  $w$  multiplied by  $p$ .

Now you know what an equation is.

Turn to page 208. (Book II)

Your answer was  $a/c = b$  or  $a = bc$ . Very good!

$a/c = b$  is equivalent to  $a = bc$ .

What is an equivalent form of the equation  $I = E/R$ ?

- (a)  $R = EI$       Turn to page 27
- (b)  $E = IR$       Turn to page 24
- (c)  $E = RI$       Turn to page 25
- (d)  $I = R/E$       Turn to page 27

Let's try another one.

What is an equivalent form of the equation  $I = E/R$ ?

- (a)  $R = EI$       Turn to page 27
- (b)  $E = IR$       Turn to page 24
- (c)  $E = RI$       Turn to page 25
- (d)  $I = R/E$       Turn to page 27

Your answer was either (a)  $3 + 4 + 7$  or (b)  $5 \times 2 + 3$ .

No! This isn't correct.

$5 \times 2 + 3$  does tell you to multiply  $5 \times 2$  and then add 3.

But, where is the equal sign? There isn't any. We must have the equal sign before we can have an equation.

$5 = 2 + 3$  is an equation. It expresses a relation between 5, 3 and 2. It says 5 equals 2 plus 3.

Another example of an equation is  $8 = 4 \times 2$ . It expresses the relation that 8 equals 4 multiplied by 2.

Which of the following is a correct equation?

(a)  $6 + 4 + 10$       Turn to page 17

(b)  $5 = 3 + 2$       Turn to page 11

(c)  $3 \times 4 + 12$       Turn to page 17

Ready? Try this one.

Which of the following is a correct equation?

(a)  $6 + 4 + 10$       Turn to page 17

(b)  $5 = 3 + 2$       Turn to page 11

(c)  $3 \times 4 + 12$       Turn to page 17

Page 10

Your answer was  $5 \times 4 = 20$ . Good!  $5 \times 4 = 20$  is an equation. It expresses a relation between 5, 4 and 20. The equation says 5 multiplied by 4 equals 20.

Turn to page 14.



Page 11

Your answer was (b)  $5 = 3 + 2$ . Good! This equation expresses a relationship between 5, 3 and 2. It says that 5 is equal to  $3 + 2$ .

Which of the following is a correct equation?

- (a)  $5 = 1 \times 5$  Turn to page 13
- (b)  $5 = 1 = 5$  Turn to page 12
- (c)  $3 + 3 + 6$  Turn to page 16

Your answer was (b)  $5 = 1 = 5$ . No! You're off the track.

$5 = 1 = 5$  certainly does have an equal sign. In fact it has two! However,  $5 = 1$  is certainly not correct. An equation must express a correct relationship between quantities.

$5 = 5$  is an equation.

$5 = 1 + 4$  is an equation which expresses the relation between 5, 1 and 4 which says 5 equals 1 plus 4.

Let's try again!

Which of the following is a correct equation?

- (a)  $5 = 5 = 3$       Turn to page 15
- (b)  $5 \times 4 = 20$       Turn to page 10
- (c)  $5 + 4 = 20$       Turn to page 15

Your answer was (a)  $5 = 1 \times 5$ . Good!  $5 = 1 \times 5$  is an equation. It says 5 equals 1 multiplied by 5.

Which of the following is an equation?

- (a)  $x = w \cdot p$  Turn to page 5
- (b)  $x + w + p$  Turn to page 23
- (c)  $3 \times 4 + 7$  Turn to page 19

You seem to have it! Try one more.

Which of the following is an equation?

(a)  $x = w \cdot p$  Turn to page 5

(b)  $x + w + p$  Turn to page 23

(c)  $3 \times 4 + 7$  Turn to page 19

Page 15

You seem to be having a little trouble. Why don't  
you tell your teacher where you are in the program.  
Maybe she can help you.

After you have seen the teacher, start on page 1.

Page 16

Your answer was (c)  $3 + 3 + 6$ . No!

Remember an equation is a relationship that has an equal sign.

$3 + 3 = 6$  is an equation.

$5 \times 4 = 20$  is an equation.

Turn to page 9.

Your answer was either (a)  $6 + 4 + 10$  or (b)  $3 \times 4 + 12$ .

No! Neither of these expressions contains an equal sign.

Remember an equation is a relation which has an equal sign.

$6 + 4 = 10$  is an equation.

$3 \times 4 = 12$  is an equation.

Is this an equation?       $5 = 1 + 4$

(a) Yes      Turn to page 18

(b) No      Turn to page 15

Your answer was (a) Yes. Very good!  $5 = 1 + 4$  is an equation.

Which of the following is an equation?

(a)  $5 \times 4 = 20$  Turn to page 10

(b)  $5 \times 2 + 3$  Turn to page 8



Your answer was  $3 \times 4 + 7$ . No! You're off the track.

$3 \times 4 + 7$  is an expression which does not contain an equal sign. Remember, an equation must contain an equal sign.

$3 + 4 = 7$  is an equation.

$a + b = c$  is an equation.

Is  $4 + 5 + 9$  an equation?

(a) Yes      Turn to page 21

(b) No      Turn to page 22

$x + w + p$  is an expression which does not contain an equal sign. Remember, an equation must contain an equal sign.

$3 + 4 = 7$  is an equation.

$a + b = c$  is an equation.

Is  $4 + 5 + 9$  an equation?

- (a) Yes      Turn to page 21
- (b) No        Turn to page 22

Your answer was (a) Yes. You are not correct.

$4 + 5 + 9$  does tell you to add  $4 + 5$  and then to add 9.  
But, where is the equal sign? There isn't any. We must  
have the equals sign before we can have an equation.

$5 = 2 + 3$  is an equation. It expresses a relation  
between 5, 2 and 3. It says 5 equals 2 plus 3.

Now turn to page 9.

Your answer was (b) No. Very good! An equation must contain an equals sign.

$4 + 5 = 9$  is an equation.

$x = w \cdot p$  is an equation.

Page 5 begins "Your answer was (a)  $x = wp$ ."

Turn to page 5.

Your answer was (b)  $x + w + p$ . No! An equation must contain an equal sign.

Turn to page 20.

Page 24

Your answer was (b)  $E = IR$ . You are doing very well.

Is the equation  $I = E/R$  also equivalent to  $E = IR$ ?

- (a) Yes      Turn to page 26
- (b) No      Turn to page 131 (Book II)

Your answer was (c)  $E = RI$ . You are doing very well!

What about the equation  $E = IR$ . Is it also equivalent to the equation  $I = E/R$ ?

- (a) Yes      Turn to page 26
- (b) No      Turn to page 131 (Book II)

Your answer was (a) Yes. Excellent! Obviously you found out that the equation  $E = IR$  and  $E = RI$  are equivalent. You must know the commutative law. This law states that the order of multiplication does not change the result. That is:

$$3 \cdot 2 = 2 \cdot 3$$

$$4 \times 3 = 3 \times 4$$

$$a \times b = b \times a$$

$$IR = RI$$

Is the product  $m \cdot n$  equal to the product  $n \cdot m$ ?

(a) Yes      Turn to page 137 (Book II)

(b) No      Turn to page 134 (Book II)



Your answer was either (a)  $R = EI$  or (d)  $I = R/E$ .

Incorrect! You almost had it. Let's run over the idea quickly.

Two equations represent equivalent statements if they both represent the same relationship.  $3 \times 2 = 6$  and  $3 = 6/2$  are equivalent statements. Both equations represent the relation between 2, 3 and 6.

Let's consider obtaining an equivalent statement by either:

- I. Multiplying both sides of the equation by the same quantity.
- II. Dividing both sides of the equation by the same quantity. (Except 0) NEVER DIVIDE BY ZERO.

In the example above we obtained an equivalent statement by dividing both sides of the equation by 2.

$$3 \times 2 = 6$$

Original equation

$$\frac{3 \times 2}{2} = \frac{6}{2}$$

Divide by 2

$$3 = 6/2$$

Here is another example using multiplication.

$$3 = 6/2$$

Original equation

$$2 \times 3 = 2 \times 6/2$$

Multiply by 2

$$2 \times 3 = 6$$

(Continued on next page)

Which equation is an equivalent statement of the equation  
 $5 \times 2 = 10$ ?

- (a)  $5 = 2 \times 10$       Turn to page 42
- (b)  $5/2 = 10/2$       Turn to page 42
- (c)  $5 = 10/2$       Turn to page 31
- (d)  $2 = 10/5$       Turn to page 32

Your answer was either (b)  $c/a = b$  or (c)  $ab = c$  or (d) don't understand. No! You don't quite have the right idea.

Here is a quick overview of the method:

Two equations represent equivalent statements if they both represent the same relationship.  $3 \times 2 = 6$  and  $3 = 6/2$  are equivalent statements. Both equations represent the relation between 2, 3 and 6.

Let's consider obtaining an equivalent statement by either:

- I. Multiplying both sides of the equation by the same quantity.
- II. Dividing both sides of the equation by the same quantity. (Except 0) NEVER DIVIDE BY ZERO.

In the example above we obtained an equivalent statement by dividing both sides of the equation by 2.

$$3 \times 2 = 6$$

Original statement

$$\frac{3 \times 2}{2} = \frac{6}{2}$$

Divide by 2

$$3 = 6/2$$

(Continued on next page)

Here is another example using multiplication.

$$3 = 6/2$$

Original equation

$$2 \times 3 = 2 \times 6/2$$

Multiply by 2

$$2 \times 3 = 6$$

Which equation is an equivalent statement of the equation

$$5 \times 2 = 10?$$

(a)  $5 = 2 \times 10$  Turn to page 42

(b)  $5/2 = 10/2$  Turn to page 42

(c)  $5 = 10/2$  Turn to page 31

(d)  $2 = 10/5$  Turn to page 32

Your answer was  $12 = 4 \times 3/3$ . No! You don't quite have the right idea.

Here is a quick overview of the method:

Two equations represent equivalent statements if they both represent the same relationship.  $3 \times 2 = 6$  and  $3 = 6/2$  are equivalent statements. Both equations represent the relation between 2, 3 and 6.

Let's consider obtaining an equivalent statement by either:

- I. Multiplying both sides of the equation by the same quantity.
- II. Dividing both sides of the equation by the same quantity. (Except 0) NEVER DIVIDE BY ZERO.

In the example above we obtained an equivalent statement by dividing both sides of the equation by 2.

$$3 \times 2 = 6$$

Original equation

$$\frac{3 \times 2}{2} = \frac{6}{2}$$

Divide by 2

$$3 = 6/2$$

(Continue on next page)

Here is another example using multiplication.

$$3 = 6/2$$

Original equation

$$2 \times 3 = 2 \times 6/2$$

Multiply by 2

$$2 \times 3 = 6$$

Which equation is an equivalent statement of the equation

$$5 \times 2 = 10?$$

(a)  $5 = 2 \times 10$  Turn to page 42

(b)  $5/2 = 10/2$  Turn to page 42

(c)  $5 = 10/2$  Turn to page 31

(d)  $2 = 10/5$  Turn to page 32

Your answer was (b) No. No! You do seem to be having a bit of trouble.

Go see your teacher and then return to page 3.

Your answer was (c)  $5 = 10/2$ . Good!

You noticed that both sides of the original equation were divided by 2.

How about answer (d)  $2 = 10/5$ . Is it also equivalent to  $5 \times 2 = 10$ ?

- (a) Yes      Turn to page 33
- (b) No        Turn to page 39



Your answer was (d)  $2 = 10/5$ . Good!

You noticed that both sides of the original equation were divided by 5.

How about the answer (c)  $5 = 10/2$ . Is it also equivalent to  $5 \times 2 = 10$ ?

(a) Yes      Turn to page 33

(b) No      Turn to page 39

Your answer was (a) Yes. Very good!

All three equations  $2 \times 5 = 10$ ,  $10/5 = 2$  and  $10/2 = 5$  are equivalent.

Which equation is equivalent to the statement  $a/b = c$ ?

- (a)  $b = ac$       Turn to page 34
- (b)  $a = bc$       Turn to page 6
- (c)  $a = c/b$       Turn to page 34

Your answer was (a)  $b = ac$  or (b)  $a = c/b$ . No!

Maybe the letters are giving you problems.

Remember, we can multiply both sides of an equation by the same quantity. This means we can multiply both sides by numbers and since letters represent numbers, we can multiply by letters too. Likewise, since we can divide both sides by numbers, we can also divide both sides by letters.

$$15/5 = 3 \quad \text{Original statement}$$

$$5 \times 15/5 = 5 \times 3 \quad \text{Multiply by 5}$$

$$15 = 5 \times 3$$

$$a/b = c \quad \text{Original statement}$$

$$b \times a/b = b \times c \quad \text{Multiply by } b$$

$$a = b \times c$$

$$E = IR \quad \text{Original statement}$$

$$\frac{E}{R} = \frac{IR}{R} \quad \text{Divide by } R \text{ (Assume } R \text{ is not equal to 0)}$$

$$E/R = I$$

Is the statement  $x/y = z$  equivalent to the statement  $x = yz$ ?

(a) Yes      Turn to page 35

(b) No      Turn to page 37

Your answer was (a) Yes. Excellent! You noticed that both sides of the first equation could be multiplied by  $y$ . This produced the second equation.

Which of the following is the statement  $d = rt$  equivalent to?

- (a)  $r = d/t$       Turn to page 36
- (b)  $r = t/d$       Turn to page 38
- (c)  $dr = t$       Turn to page 38

Page 36

Your answer was (a)  $r = d/t$ . Very good! You seem to have the idea of equivalent statements.

Turn to page 3.

Your answer was (b) No. You are incorrect.

You seem to have the idea in terms of numbers. Let's try a little review of working with letters.

Multiplying by letters is much like multiplying by numbers. For example, suppose we want to multiply 3 by  $7/3$ .  $3 \times 7/3 = 7$ .

Now, consider the same thing only using letters. Multiply  $a$  by  $b/a$ .  $a \times b/a = b$ .

What is the result of the problem  $c \times a/c = ?$

- (a)  $a$  Turn to page 105
- (b)  $c$  Turn to page 44

Your answer was either (b)  $r = t/d$  or (c)  $dr = t$ .

No! You are not right.

You seem to have the idea in terms of numbers. Let's try a little review of working with letters.

Multiplying by letters is much like multiplying by numbers. For example, suppose we want to multiply 3 by  $7/3$ .  $3 \times 7/3 = 7$ .

Now, consider the same thing only using letters. Multiply  $a$  by  $b/a$ .  $a \times b/a = b$ .

What is the result of the problem  $c \times a/c = ?$

- (a)  $a$       Turn to page 105
- (b)  $c$       Turn to page 44

Your answer was (b) No. No! This isn't right.

Our original statement was:  $5 \times 2 = 10$ .

There are two equivalent statements we can make by division. One statement is obtained by dividing both sides of the original equation by 2. Dividing by 2 we obtain  $5 = 10/2$ . A second equivalent statement is obtained by dividing both sides of the original equation by 5. Dividing by 5 we obtain  $2 = 10/5$ .

Three equivalent forms of the statement  $5 \times 2 = 10$  are:

$$(5 \times 2 = 10) \quad (5 = 10/2) \quad (2 = 10/5)$$

Is the statement  $4 \times 2 = 8$  equivalent to the statement  $4 = 8/2$ ?

- (a) Yes      Turn to page 40
- (b) No       Turn to page 41



Your answer is (a) Yes. Very good!

Are all three of the following statements equivalent?

$$(2 = 10/5) \quad (5 \times 2 = 10) \quad (5 = 10/2)$$

(a) Yes      Turn to page 33

(b) No      Turn to page 41

Page 41

Your answer was (b) No. No! You don't quite have it.

Turn to page 45 for further explanation.

Your answer was either (a)  $5 = 2 \times 10$  or (b)  $5/2 = 10/2$ .

No!

Let's look at a very short summary of forming equivalent statements. Remember, two ways of forming equivalent statements are:

- I. Multiply both sides of the equation by the same quantity.

$$6/2 = 3$$

Original statement

$$2 \times 6/2 = 2 \times 3$$

Multiply by 2

$$6 = 2 \times 3$$

- II. Divide both sides of the equation by the same quantity. (Except 0)

$$6 = 2 \times 3$$

Original statement

$$\frac{6}{3} = \frac{2 \times 3}{3}$$

Divide by 3

$$6/3 = 2$$

Which of the following is equivalent to  $14/2 = 7$ ?

- (a)  $14 = 7/2$  Turn to page 46  
(b)  $14 = 2 \times 7$  Turn to page 43  
(c)  $14 = 7 \times 2$  Turn to page 43  
(d)  $2 \times 14 = 2 \times 7$  Turn to page 46

Your answer was either (b)  $14 = 2 \times 7$  or (c)  $14 = 7 \times 2$ .

Very good! Both  $14 = 2 \times 7$  and  $14 = 7 \times 2$  are equivalent to  $14/2 = 7$ .

The fact that  $2 \times 7 = 7 \times 2$  shows the COMMUTATIVE LAW for multiplication. This means the order of multiplication does not make any difference.

Which of the following statements are equivalent to

$10 = 2 \times 5$ ?

(a)  $10 \times 2 = 5$       Turn to page 46

(b)  $5/10 = 2$       Turn to page 46

(c)  $5 = 10/2$       Turn to page 31

(d)  $2 = 10/5$       Turn to page 32

Your answer was (b) c. Incorrect!

Now really! We have been through this before.

$c \times a/c$  can be written  $\frac{c \times a}{c}$

$\frac{c \times a}{c}$  can be written  $\frac{c}{c} \times a$

and  $c/c \times a$  is equal to  $a$ .

This is just like the problem  $3 \times 4/3 = 4$

Now, go to page 106.

Now, let us look at the formation of equivalent equations more carefully.

Often there are many correct ways to do a mathematics problem. It is important to think about the problem and to try something. What you try may not work, but at least you will gain more information upon which to base your next try.

To form an equivalent equation, the first thing we might try is to multiply both sides of the equation by the same quantity. If we multiply both sides of an equation by the same quantity, we always produce an equivalent equation. The new statement may not be the exact statement we are looking for. But, the new statement is equivalent to the original. For example, we want to decide whether  $12/4 = 3$  is equivalent to  $12 = 4 \times 3$ . Let's first try multiplying both sides of the original statement by 3.

$12/4 = 3$	Original statement
$2 \times 12/4 = 3 \times 2$	Multiply by 2
$12/2 = 3 \times 2$	an equivalent statement

$12/4 = 3$  and  $12/2 = 3 \times 2$  are equivalent statements.

However,  $12/2 = 3 \times 2$  is not the equivalent statement we were looking for.

Turn to page 48.

Your answer was not correct. Let's see why.

Often there are many correct ways to do a mathematics problem. It is important to think about the problem and to try something. What you try may not work, but at least you will gain more information upon which to base your next try.

To form an equivalent equation, the first thing we might try is to multiply both sides of the equation by the same quantity. If we multiply both sides of an equation by the same quantity, we always produce an equivalent equation. The new statement may not be the exact statement we are looking for. But, the new statement is equivalent to the original. For example, we want to decide whether  $12/4 = 3$  is equivalent to  $12 = 4 \times 3$ . Let's first try multiplying both sides of the original statement by 3.

$$12/4 = 3$$

Original statement

$$2 \times 12/4 = 3 \times 2$$

Multiply by 2

$$12/2 = 3 \times 2$$

an equivalent statement

$12/4 = 3$  and  $12/2 = 3 \times 2$  are equivalent statements.

However,  $12/2 = 3 \times 2$  is not the equivalent statement we were looking for.

Turn to page 48.

Your answer was (a) divide by 4. Incorrect!

Often there are many correct ways to do a mathematics problem. It is important to think about the problem and to try something. What you try may not work, but at least you will gain more information upon which to base your next try.

To form an equivalent equation, the first thing we might try is to multiply both sides of the equation by the same quantity. If we multiply both sides of an equation by the same quantity, we always produce an equivalent equation. The new statement may not be the exact statement we are looking for. But, the new statement is equivalent to the original. For example, we want to decide whether  $12/4 = 3$  is equivalent to  $12 = 4 \times 3$ . Let's first try multiplying both sides of the original statement by 3.

$$12/4 = 3$$

Original statement

$$2 \times 12/4 = 3 \times 2$$

Multiply by 2

$$12/2 = 3 \times 2$$

an equivalent statement

$12/4 = 3$  and  $12/2 = 3 \times 2$  are equivalent statements.

However,  $12/2 = 3 \times 2$  is not the equivalent statement we are looking for.

Turn to page 48.



Let's try again. We want to know if  $12/4 = 3$  is equivalent to the statement  $12 = 4 \times 3$ .

$$12/4 = 3 \quad \text{Original statement}$$

$$3 \times 12/4 = 3 \times 3 \quad \text{Multiply by 3}$$

We are not on the right track. We want the right side of the equation to be  $4 \times 3$ .

Let's start again.  $12/4 = 3$  is the original statement.

We know multiplying by 3 does not produce the results we want. Let's try multiplying by 4.

$$4 \times 12/4 = 4 \times 3 \quad \text{Multiply by 4}$$

$$12 = 4 \times 3$$

You probably have noticed that we multiply by one of the quantities in the original equation. In an equation like  $12/4 = 3$  there are only three choices in what to multiply by: either 12, 4 or 3. If one of these does not result in the equivalent statement for which you are looking, then a single multiplication by any whole number will not produce the equivalent statement.

We are starting with the original statement  $10/2 = 5$ .

We want to form the equivalent statement  $10 = 2 \times 5$ .

What quantity should we multiply the original statement by?

(a) 10      Turn to page 92

(b) 5        Turn to page 93

(c) 2        Turn to page 49

Your answer was (c) 2. Correct!

$$10/2 = 5$$

Original statement

$$2 \times 10/2 = 2 \times 5$$

Multiply by 2

$$10 = 2 \times 5$$

Try another.

What do you multiply  $20/4 = 5$  by to obtain the equivalent statement  $20 = 4 \times 5$ ?

(a) 20

Turn to page 92

(b) 5

Turn to page 93

(c) 4

Turn to page 50

(d) None of the above

Turn to page 92

Your answer was (c) 4. Excellent, you are right!

Sometimes it is not possible to obtain the equivalent statement we want by multiplication.

A second way of constructing equivalent statements is by division. To form an equivalent statement we can: Divide both sides of the equation by the same quantity. (Remember, NEVER DIVIDE BY ZERO)

For example, we want to decide whether  $15 = 5 \times 3$  is equivalent to  $15/5 = 3$ . Let's first try dividing both sides of the original statement by 5.

$$15 = 5 \times 3$$

Original statement

$$\frac{15}{5} = \frac{5 \times 3}{5}$$

Divide by 5

$$15/5 = 3$$

We can say  $15/5 = 3$  and  $15 = 5 \times 3$  are equivalent statements.

Turn to page 51.

Suppose we want to know if  $7 = 7 \times 1$  is equivalent to the statement  $7/1 = 7$ ? We might try dividing by 7.

$$7 = 7 \times 1$$

Original statement

$$\frac{7}{7} = \frac{7 \times 1}{7}$$

Divide by 7

$$7/7 = 1$$

We are not on the right track. Although the new statement is equivalent to the original, the new statement is not the one we are looking for. We want the right side of the new statement to be 7. You might think that we should try dividing the original statement by 1.

$$7 = 7 \times 1$$

Original statement

$$\frac{7}{1} = \frac{7 \times 1}{1}$$

Divide by 1

$$7/1 = 7$$

Very good! You helped figure out our problem. We have shown that  $7 = 7 \times 1$  and  $7/1 = 7$  are equivalent statements

By now you have probably noticed that we divide by one of the quantities in the original equation. If one of these does not produce the equivalent statement you are looking for, then a single division by any whole number will not produce the equivalent statement.

(Continued on next page)

Page 51 (Cont.)

We are starting with the original statement  $6 = 3 \times 2$ .

We want to form the equivalent statement  $6/2 = 3$ . By what quantity should we divide the original statement?

- (a) 6      Turn to page 76
- (b) 3      Turn to page 77
- (c) 2      Turn to page 52

Your answer was (c) 2. Very good!

Repeating an example:

$$6 = 2 \times 3$$

Original statement

$$\frac{6}{2} = \frac{2 \times 3}{2}$$

Divide by 2

$$6/2 = 3$$

What do you divide  $30 = 6 \times 5$  by to obtain the equivalent statement  $30/5 = 6$ ?

- (a) 30      Turn to page 78
- (b) 6       Turn to page 76
- (c) 5       Turn to page 53

Your answer was (c) 5. Correct, you are doing very well!

What do you divide  $15/3 = 5$  by to obtain the equivalent statement  $15 = 5 \times 3$ ?

- |                       |                            |
|-----------------------|----------------------------|
| (a) 15                | Turn to page 226 (Book II) |
| (b) 5                 | Turn to page 226 (Book II) |
| (c) 3                 | Turn to page 226 (Book II) |
| (d) None of the above | Turn to page 54            |

Your answer was (d) none of the above. Excellent!

In review; there are two ways we have learned to form equivalent statements.

- I. Multiply both sides of the original statement by the same quantity.
- II. Divide both sides of the original statement by the same quantity. (NEVER DIVIDE BY ZERO)

Suppose we start with the original statement  $14 = 2 \times 7$ .

We want to obtain the equivalent statement  $14/7 = 2$ . We should:

- (a) divide the original statement by 2      Turn to page 69
- (b) divide the original statement by 7      Turn to page 55
- (c) multiply the original statement by 2      Turn to page 73
- (d) multiply the original statement by 7      Turn to page 73



Your answer was (b) divide the original statement by 7.

Very good!

Is the statement  $6 = 6 \times 1$  equivalent to the statement  
 $6/6 = 1$ ?

- (a) Yes      Turn to page 56
- (b) No       Turn to page 66

Your answer was (a) Yes. Congratulations! You are right.

Is the statement  $8/2 = 4$  equivalent to the statement  
 $8 = 4 \times 2$ ?

- (a) Yes      Turn to page 57
- (b) No      Turn to page 58

Page

Your answer was (a) Yes. Excellent!

Now, turn to page 3 and continue.

Your answer was (b) No. No! You really don't mean that.

We began with  $8/2 = 4$  as the original statement. Now, we can either multiply or divide by 8, 4 or 2. If we multiply both sides of the equation by 2 we obtain:

$$2 \times 8/2 = 4 \times 2 \quad \text{Multiply by 2}$$

$$8 = 4 \times 2$$

Which of the following is equivalent to  $9/3 = 3$ ?

(a)  $9 = 3 \times 3$       Turn to page 59

(b)  $3 \times 9 = 3$       Turn to page 61

Your answer was (a)  $9 = 3 \times 3$ . Very good! You're back on the right track.

Which of the following is equivalent to  $15 = 3 \times 5$ ?

(a)  $5/3 = 15$       Turn to page 62

(b)  $15/5 = 3$       Turn to page 60

Page 60

Your answer was (b)  $15/5 = 3$ . Right!

Now, you have the correct idea.

Turn to page 3.

Your answer was (b)  $3 \times 9 = 3$ . No!

Now, stop and think a minute. Is your answer to that last question a correct statement?

Of course it isn't!

You were doing so well, probably just a brief review will get you back on the track.

Remember to produce an equivalent statement we can:

- I. Multiply both sides of the equation by the same quantity.
- II. Divide both sides of the equation by the same quantity. (NEVER DIVIDE BY ZERO)

Which statement is equivalent to the statement  $21/7 = 3$ ?

- |                       |                 |
|-----------------------|-----------------|
| (a) $21 = 7 \times 3$ | Turn to page 63 |
| (b) $21 = 3 \times 7$ | Turn to page 63 |
| (c) $7/3 = 21$        | Turn to page 65 |
| (d) $7/21 = 3$        | Turn to page 65 |

Your answer was (a)  $5/3 = 15$ . No!

Now, stop and think a minute. Is your answer to that last question a correct statement?

Of course it isn't!

You were doing so well, probably just a brief review will get you back on the track.

Remember, to produce an equivalent statement we can:

- I. Multiply both sides of the equation by the same quantity.
- II. Divide both sides of the equation by the same quantity. (NEVER DIVIDE BY ZERO)

Which statement is equivalent to the statement  $21/7 = 3$ ?

- (a)  $21 = 7 \times 3$       Turn to page 63
- (b)  $21 = 3 \times 7$       Turn to page 63
- (c)  $7/3 = 21$           Turn to page 65
- (d)  $7/21 = 3$           Turn to page 65



Your answer was either (a)  $21 = 7 \times 3$  or (b)  $21 = 3 \times 7$ .

Correct!

You probably noticed that  $7 \times 3$  and  $3 \times 7$  are equivalent. They both are examples of the COMMUTATIVE LAW of multiplication. That is, the order of multiplication does not make any difference in the result.

Which of the following is equivalent to  $18 = 6 \times 3$ ?

- |                       |                 |
|-----------------------|-----------------|
| (a) $6/18 = 3$        | Turn to page 64 |
| (b) $3/18 = 6$        | Turn to page 64 |
| (c) both a and c      | Turn to page 64 |
| (d) none of the above | Turn to page 54 |

Page 64

Your answer was either (a)  $6/18 = 3$ , (b)  $3/18 = 6$  or  
(c) both a and c. No!

Let's review the idea of equivalence again.

Turn to page 92.

Page 65

Your answer was either (c)  $7/3 = 21$  or (d)  $7/21 = 3$ .

No!

Let's review the idea of equivalence again.

Turn to page 92.

Your answer was (b) No. Incorrect! Maybe the 1 confused you a bit. There are many ways of writing 1:  $6/6$  is one way.

Let's look at the problem again.

$$6 = 6 \times 1$$

Original statement

$$6/6 = 1$$

Divide by 6

Do you think you would like to review more or go on with the program?

- (a) More review please      Turn to page 228 (Book II)
- (b) Go on with program      Turn to page 67

Your response was (b) Go on with the program.

Very courageous of you!

Which of the following is equivalent to the statement

$$5/5 = 1?$$

- (a)  $1/5 = 1$       Turn to page 227 (Book II)
- (b)  $5/1 = 1$       Turn to page 227 (Book II)
- (c)  $5 = 1 \times 5$       Turn to page 68

Your answer was (c)  $5 = 1 \times 5$ . Correct!

Is the statement  $10/2 = 5$  equivalent to the statement  
 $10 = 2 \times 5$ ?

- (a) Yes      Turn to page 57
- (b) No        Turn to page 58

Your answer was (a) divide the original statement by 2.  
No! You probably didn't read the question carefully enough.

Let's look it over from another point of view.

$$14 = 2 \times 7$$

Original statement

We want to obtain the equivalent statement  $14/7 = 2$ .

You were correct in saying we should divide the original equation.

Now, doesn't it look like we should divide by 7?

(a) Yes      Turn to page 70

(b) No      Turn to page 72

Your answer was (a) Yes. Right! I knew you'd get it.

Suppose we start with the original statement  $21 = 3 \times 7$ .

We want to obtain the equivalent statement  $21/7 = 3$ .

We should:

- (a) divide the original statement by 3      Turn to page 71
- (b) divide the original statement by 7      Turn to page 55
- (c) multiply the original statement by 3      Turn to page 74
- (d) multiply the original statement by 7      Turn to page 74



Your answer was (a) divide the original statement by 3. No!

Let's go over the idea of dividing once more. To form an equivalent statement we can: divide both sides of the equation by the same quantity (NEVER DIVIDE BY ZERO).

For example, we want to decide whether  $18 = 6 \times 3$  is equivalent to  $18/6 = 3$ .

$$18 = 6 \times 3$$

Original statement

Let's try dividing by 6.

$$\frac{18}{6} = \frac{6 \times 3}{6}$$

Divide by 6

$$18/6 = 3$$

We see that division by 6 does produce  $18/6 = 3$ . Therefore,  $(18 = 6 \times 3)$  and  $(18/6 = 3)$  must be equivalent statements.

What must we divide  $12 = 6 \times 2$  by to obtain the equivalent statement  $12/2 = 6$ ?

- (a) 12     Turn to page 75
- (b) 6     Turn to page 75
- (c) 2     Turn to page 52

Your answer was (b) No. This isn't correct!

Let's go over the idea of dividing once more. To form an equivalent statement we can: divide both sides of the equation by the same quantity (NEVER DIVIDE BY ZERO).

For example, we want to decide whether  $18 = 6 \times 2$  is equivalent to  $18/6 = 3$ .

$$18 = 6 \times 3$$

Original statement

Let's try dividing by 6.

$$\frac{18}{6} = \frac{6 \times 3}{6}$$

Divide by 6

$$18/6 = 3$$

We see that division by 6 does produce  $18/6 = 3$ . Therefore,  $(18 = 6 \times 3)$  and  $(18/6 = 3)$  must be equivalent statements.

What must we divide  $12 = 6 \times 2$  by to obtain the equivalent statement  $12/2 = 6$ .

- (a) 12      Turn to page 75
- (b) 6        Turn to page 75
- (c) 2        Turn to page 52

Your answer was either (c) multiply the original statement by 2 or (d) multiply the original statement by 7.  
No!

You seem pretty determined to multiply. That won't work in situations such as this. In fact we should divide by 7.

$$21 = 3 \times 7$$

Original statement

$$\frac{21}{7} = \frac{3 \times 7}{7}$$

Divide by 7

$$21/7 = 3$$

Desired equivalent

For another example in using division to obtain an equivalent statement turn to page 79.

Your answer was either (c) multiply the original statement by 3 or (d) multiply the original statement by 7.  
No!

You seem pretty determined to multiply. That won't work in situations such as this. In fact we should divide by 7.

$$21 = 3 \times 7$$

Original statement

$$\frac{21}{7} = \frac{3 \times 7}{7}$$

Divide by 7

$$21/7 = 3$$

Desired equivalent

For another example in using division to obtain an equivalent statement turn to page 79.

Page 75

Your answer was (a) 12 or (b) 6. Incorrect!

You seem to be having a little difficulty deciding what to divide by.

Turn to page 79.

Page 76

Your answer was 6. Incorrect!

You seem to be having a little difficulty deciding what to divide by.

Turn to page 79.

Page 77

Your answer was 3. Incorrect!

You seem to be having a little difficulty in deciding what to divide by.

Turn to page 79.

Page 78

Your answer was (a) 30. Incorrect!

You seem to be having a little difficulty in deciding what to divide by.

Turn to page 79.



Let's see if we can decide what to divide by:

Remember that so far we have obtained equivalent statements by one of two ways:

- I. Divide both sides of the equation by the same quantity (NEVER DIVIDE BY ZERO).
- II. Multiply both sides of the equation by the same quantity.

Think of the method of division. In every case we have looked at, we have performed only one division.

$$12 = 4 \times 3$$

Original statement

$$\frac{12}{4} = \frac{4 \times 3}{4}$$

Divide by 4

$$12/4 = 3$$

The only division in this problem was division by 4.

We want to see whether  $15 = 5 \times 3$  is equivalent to the statement  $15/3 = 5$ .

$$15 = 5 \times 3$$

Original statement

In fact for a minute think of only one side of the two statements.

15 left side of the original statement

$15/3$  left side of the new statement

You will notice that the 15 has been divided by 3.

(Continue on next page)

Now, we see that 3 is the number we want to divide by.

$$15 = 5 \times 3$$

Original statement

$$\frac{15}{3} = \frac{5 \times 3}{3}$$

Divide both sides of the equation by 3.

$$15/3 = 5$$

New statement

We see that  $(15 = 5 \times 3)$  is equivalent to  $(15/3 = 5)$ .

Turn to page 80.

Let's look at another. We want to decide whether  
 $18 = 6 \times 3$  is equivalent to the statement  $18/6 = 3$ .

To decide what to do we shall look at the left hand  
sides of the two statements.

18      Original statement

18/6    New statement

Starting with the left side of the statement (18) did  
we divide by 6 to obtain the left side of the new  
statement (18/6)?

(a) Yes      Turn to page 81

(b) No      Turn to page 91

Your answer was (a) Yes. Very good! We do divide 18 by 6 to obtain  $18/6$ .

Now, let's try another.  $6 \times 3 = 18$  is the original statement. The new equivalent statement is  $6 = 18/3$ .

Look at the right side of the original statement and then at the right side of the new statement.

What did we divide 18 by to obtain  $18/3$ ?

- (a) 18      Turn to page 89
- (b) 6       Turn to page 89
- (c) 3       Turn to page 82

Your answer was (c) 3. Right! Now you have it. Let's try one more.

We want to show that the statement  $6 \times 3 = 18$  is equivalent to the statement  $6 = 18/3$ . It should be remembered that when we divide, we must divide both sides of the equation by the same quantity.

What do you divide  $6 \times 3 = 18$  by to obtain the equivalent statement  $6 = 18/3$ ?

- |                               |                 |
|-------------------------------|-----------------|
| (a) 18                        | Turn to page 88 |
| (b) 6                         | Turn to page 88 |
| (c) 3                         | Turn to page 83 |
| (d) I haven't any idea. HELP! | Turn to page 87 |

Your answer was (c) 3. Excellent, you are on the right track.

Suppose we start with the original equation  $6 = 2 \times 3$ .  
What do we divide the original statement by to obtain  
the equivalent statement  $6/2 = 3$ ?

- (a) 6      Turn to page 85
- (b) 3      Turn to page 86
- (c) 2      Turn to page 52

Your answer was (c) 2.

Suppose we start with the original equation  $6 = 2 \times 3$ .  
What do we divide the original statement by to obtain  
the equivalent statement  $6/2 = 3$ ?

- (a) 6      Turn to page 85
- (b) 3      Turn to page 86
- (c) 2      Turn to page 52

Your answer was (a) 6. No! Now really, how did you get that!

Our original statement was  $6 = 2 \times 3$ . Our new equivalent statement was  $6/2 = 3$ . Now, certainly we divide 6 by 2 to obtain 6/2. We always divide both sides of the equation by the same quantity. So here's what we did.

$$6 = 2 \times 3$$

Original statement

$$\frac{6}{2} = \frac{2 \times 3}{2}$$

Divide both sides of the equation by 2.

$$6/2 = 3$$

New equivalent statement

What must we divide both sides of the original equation  $18 = 6 \times 3$  by to obtain the equivalent statement  $18/3 = 6$ ?

- (a) 18      Turn to page 87
- (b) 6        Turn to page 87
- (c) 3        Turn to page 82



Your answer was (b) 3. No'

Our original statement was  $6 = 2 \times 3$ . Our new equivalent statement was  $6/2 = 3$ . Now, certainly we divide 6 by 2 to obtain 6/2. We always divide both sides of the equation by the same quantity. So here's what we did.

$$6 = 2 \times 3$$

Original statement

$$\frac{6}{2} = \frac{2 \times 3}{2}$$

Divide both sides of the equation by 2.

$$6/2 = 3$$

New equivalent statement

What must we divide both sides of the original equation

$18 = 6 \times 3$  by to obtain the equivalent statement  $18/3 = 6$ ?

- (a) 18      Turn to page 87
- (b) 6        Turn to page 87
- (c) 3        Turn to page 82

Now, you have me stumped.

Tell your teacher where you are having trouble. After  
a nice human explanation return to page 51.

Your answer was (a) 18 or (b) 6. No!

I thought you had it. Here's how we do the problem.

$$6 \times 3 = 18$$

Original statement

$$\frac{6 \times 3}{3} = \frac{18}{3}$$

Divide by 3

$$6 = 18/3$$

We noticed that we divided the right side of the original statement (18) by 3 to obtain 18/3. Therefore, we must divide both sides of the equation by 3.

It might help you to briefly review how we decide what to divide by.

Turn to page 79.

Your answer was either (a) 18 or (b) 6. No!  
Maybe you did not understand the question.

We started with 18. We want to obtain  $18/3$ .

What can we divide 18 by to obtain  $18/3$ ?

- (a) 3      Turn to page 90
- (b) 18     Turn to page 87

Page 90

Your answer was (a) 3. Right! I knew you'd get it.

Turn to page 80.

Your answer was (b) No. Incorrect! Maybe you did not read the question correctly.

We began with 18. We divided and obtained  $18/6$ .

Did we divide 18 by 6 to obtain  $18/6$ ?

- (a) Yes      Turn to page 81
- (b) No        Turn to page 87
- (c) HELP     Turn to page 87

Your answer is incorrect! Let's look at some examples.

Remember the method we are using to obtain equivalent statements: multiply both sides of the equation by the same quantity.

$$21/3 = 7$$

Original statement

$$3 \times 21/3 = 3 \times 7$$

Multiply both sides of equation by 3

$$21 = 3 \times 7$$

The only multiplication in this problem was multiplying by 3.

Suppose we want to see whether  $16/2 = 8$  is equivalent to the statement  $16 = 2 \times 8$ .

$$16/2 = 8$$

Original statement

Now look at the new equation we want to form.

$$16 = 2 \times 8$$

In fact for a minute consider only one side of the two statements.

16/2 left side of the original statement

16 left side of the new statement

You will notice that 16/2 of the original statement has been multiplied by 2. Now we see that 2 is the number we want to multiply by.

(Continue on next page)

Page 92 (Cont.)

$$16/2 = 8$$

Original statement

$$2 \times 16/2 = 2 \times 8$$

Multiply both sides of  
the equation by 2.

$$16 = 2 \times 8$$

New statement

We see that  $(16/2 = 8)$  is equivalent to  $(16 = 2 \times 8)$ .

Turn to page 94.



Your answer was 5. That is incorrect!

Remember the method we are using to obtain equivalent statements: multiply both sides of the equation by the same quantity.

$$21/3 = 7$$

Original statement

$$3 \times 21/3 = 3 \times 7$$

Multiply both sides of the equation by 3

$$21 = 3 \times 7$$

New statement

The only multiplication in this problem was multiplying by 3.

Suppose we want to see whether  $16/2 = 8$  is equivalent to the statement  $16 = 2 \times 8$ .

$$16/2 = 8$$

Original statement

Now look at the new equation we want to form.

$$16 = 2 \times 8$$

In fact for a minute consider only one side of the two statements.

16/2 left side of the original statement

16 left side of the new statement

You will notice that 16/2 of the original statement has been multiplied by 2. Now we see that 2 is the number we want to multiply by.

(Continued on next page)

Page 93 (Cont.)

$$16/2 = 8$$

Original statement

$$2 \times 16/2 = 2 \times 8$$

Multiply both sides of  
the equation by 2

$$16 = 2 \times 8$$

New statement

We see that  $(16/2 = 8)$  is equivalent to  $(16 = 2 \times 8)$ .

Turn to page 94.

Let's look at another. We want to decide whether the statement  $18/2 = 9$  is equivalent to the statement  $18 = 2 \times 9$ .

To decide what to do we shall look at the left hand side of the two statements:

$18/2$  original statement

18 new statement

Starting with the left side of the original statement ( $18/2$ ) did we multiply by 2 to obtain the left side of the new statement (18)?

- (a) Yes      Turn to page 95
- (b) No        Turn to page 104

Your answer was (a) Yes. Very good! We did multiply  $18/2$  by 2 to obtain 18.

Let's try another.  $3 = 15/5$  is the original statement.

The new equivalent statement is  $3 \times 5 = 15$ .

Look at the right side of the original statement and then look at the right side of the new statement.

What did we multiply  $15/5$  by to obtain 15?

- (a) 15      Turn to page 102
- (b) 5        Turn to page 96
- (c) 3        Turn to page 102

Your answer was (b) 5. Correct! You've got it now.

We now want to show that the statement  $5 = 20/4$  is equivalent to the statement  $4 \times 5 = 20$ . Remember, when we multiply we must multiply both sides of the equation by the same quantity.

What do you multiply  $5 = 20/4$  by to obtain the equivalent statement  $4 \times 5 = 20$ ?

- |                              |                  |
|------------------------------|------------------|
| (a) 20                       | Turn to page 101 |
| (b) 5                        | Turn to page 101 |
| (c) 4                        | Turn to page 97  |
| (d) I haven't any idea. HELP | Turn to page 100 |

Your answer was (c) 4. Excellent, you are on the right track.

Suppose we start with the original equation  $8/2 = 4$ . What do we multiply the original equation by to obtain the equivalent statement  $8 = 2 \times 4$ ?

- (a) 8      Turn to page 98
- (b) 4      Turn to page 99
- (c) 2      Turn to page 84

Your answer was (a) 8. No! How did you ever get that!

Our original statement was  $8/2 = 4$ . Our new equivalent statement was  $8 = 2 \times 4$ . Now, certainly we multiply  $8/2$  by 2 to obtain 8. We always multiply both sides of the equation by the same quantity. So, here's what we did.

$$8/2 = 4$$

Original statement

$$2 \times 8/2 = 2 \times 4$$

Multiply both sides of the equation by 2

$$8 = 2 \times 4$$

New equivalent statement

What must we multiply both sides of the original equation  $10/5 = 2$  by to obtain the equivalent statement  $10 = 5 \times 2$ ?

- (a) 10     Turn to page 100
- (b) 5     Turn to page 96
- (c) 2     Turn to page 100

Your answer was (b) 4. No!

Our original statement was  $8/2 = 4$ . Our new equivalent statement was  $8 = 2 \times 4$ . Now, certainly we multiply  $8/2$  by 2 to obtain 8. We always multiply both sides of the equation by the same quantity. So, here's what we did.

$$8/2 = 4$$

Original statement

$$2 \times 8/2 = 2 \times 4$$

Multiply both sides of the equation by 2

$$8 = 2 \times 4$$

New equivalent statement

What must we multiply both sides of the original equation  $10/5 = 2$  by to obtain the equivalent statement  $10 = 5 \times 2$ ?

- (a) 10      Turn to page 100
- (b) 5        Turn to page 96
- (c) 2        Turn to page 100



Page 100

Oh well, everyone has problems sometime.

Tell your teacher where you are having trouble. After  
a nice human explanation return to page 48.

Your answer was (a) 20 or (b) 5. No!

I thought you had it. Here's how we do the problem.

$$5 = 20/4 \quad \text{Original statement}$$

$$4 \times 5 = 4 \times 20/4 \quad \text{Multiply by 4}$$

$$4 \times 5 = 20$$

We noticed that we multiplied the right side of the original statement  $(20/4)$  by 4 to obtain  $(20)$ .

Therefore, we must multiply both sides of the equation by 4.

It might help you to briefly review how we decide what to multiply by.

Turn to page 94.

Your answer was either (a) 15 or (b) 3. No!

Perhaps you did not understand the question. We started with  $15/5$ . We want to obtain 15.

What can we multiply  $15/5$  by to obtain 15?

- (a) 15      Turn to page 100
- (b) 5        Turn to page 103

Page 103

Your answer was (b) 5. Good!

Turn to page 94.

Your answer was (b) No. Incorrect!

Maybe you did not read the question correctly.

We began with  $18/2$ . We multiplied and obtained 18.

Did we multiply  $18/2$  by 2 to obtain 18?

- (a) Yes      Turn to page 95
- (b) No      Turn to page 100

Your answer was (a) a. Correct!

Let's try another.

What do we obtain as a result if we multiply  $x/y$  by  $y$ ?

- (a)  $y$  Turn to page 118 (Book 11)
- (b)  $x$  Turn to page 107

Let's try another.

What do we obtain as a result if we multiply  $x/y$  by  $y$ ?

- (a)  $y$       Turn to page 118 (Book II)
- (b)  $x$       Turn to page 107

Your answer was (b) x. Very good!

Dividing by letters is very similar to dividing by numbers. For example, suppose we divide 3 by 4.

$$3 \div 4 = 3/4$$

Now, let's divide a by b.

$$a \div b = a/b$$

How would you show x divided by y?

- (a)  $x/y$       Turn to page 108
- (b)  $y/x$       Turn to page 120 (Book II)



Page 108

Your answer was (a)  $x/y$ . You are correct!

What do we obtain if we divide  $b$  by  $c$ ?

- (a)  $b/c$       Turn to page 109
- (b)  $c/b$       Turn to page 124 (Book II)

Your answer was (a) b/c. Correct again!

Suppose we multiply both sides of the statement  
 $x = y/w$  by  $w$ . What do we obtain?

- (a)  $wx = y$       Turn to page 110
- (b)  $wx = y/w$       Turn to page 112
- (c)  $x = y$       Turn to page 112

Your answer was (a)  $wx = y$ . Excellent!

We began with the statement:  $x = y/w$

We multiplied by  $w$ :  $wx = w \cdot y/w$

We obtained the equivalent statement:  $wx = y$

Can we say that the statement  $c/y = b$  is equivalent  
to the statement  $c = by$ ?

- (a) Yes      Turn to page 35
- (b) No       Turn to page 111

Page 111

Your answer was (b) No. Incorrect!

Maybe you need a little more review.

If you would like more review in terms of numbers:

Turn to page 45

If you would like more review in terms of letters:

Turn to page 113

Your answer was (b)  $wx = y/w$  or (c)  $x = y$ . No!  
Remember, multiply both sides of the equation by  
the same quantity.

We began with:  $x = y/w$

We multiplied both sides of the equation by  $w$ :

$$wx = w \cdot y/w$$

$$wx = y$$

Let's try again. What do we obtain if we multiply both  
sides of the statement  $r = d/t$  by  $t$ ?

- (a)  $rt = d$  Turn to page 114
- (b)  $rt = d/t$  Turn to page 127 (Book II)
- (c)  $r = d$  Turn to page 128 (Book II)
- (d) I really don't understand.  
Turn to page 45

Remember, multiply both sides of the equation by the same quantity.

We began with:  $x = y/w$

We multiplied both sides of the equation by  $w$ :

$$wx = w \cdot y/w$$

$$wx = y$$

Let's try again. What do we obtain if we multiply both sides of the statement  $r = d/t$  by  $t$ ?

- (a)  $rt = d$       Turn to page 114
- (b)  $rt = d/t$       Turn to page 127 (Book II)
- (c)  $r = d$       Turn to page 128 (Book II)
- (d) I really don't understand.  
Turn to page 45

Your answer was (a)  $rt = d$ . Very good!

What statement do we obtain if we multiply both sides of the equation  $I = E/R$  by  $R$ ?

- (a)  $IR = E/R$       Turn to page 129 (Book II)
- (b)  $IR = E$       Turn to page 115 (Book II)